Mixed Branes Interaction in Compact Spacetime

H. Arfaei and D. Kamani

Institute for Studies in Theoretical Physics and Mathematics

Tehran P.O.Box: 19395-5531, Iran

and

Department of physics, Sharif University of Technology P.O.Box: 11365-9161

e-mail:arfaei@theory.ipm.ac.ir

e-mail:kamani@theory.ipm.ac.ir

Abstract

We present a general description of two mixed branes interactions. For this we consider two mixed branes with dimensions p_1 and p_2, in external field $B_{\mu\nu}$ and arbitrary gauge fields $A^{(1)}_{\alpha_1}$ and $A^{(2)}_{\alpha_2}$ on the world volume of them, in spacetime in which some of its directions are compactified on circles with different radii. Some examples are considered to clear these general interactions. Finally contribution of the massless states on the interactions is extracted. Closed string with mixed boundary conditions and boundary state formalism, provide useful tools for calculation of these interactions.

PACS numbers:11.25.-w; 11.25.Mj; 11.30.pb

1 Introduction

A way of describing D-branes is boundary state formalism [1, 2, 3, 4, 5, 6, 7]. The boundary state can be interpreted as a source for a closed string emitted by a D-brane. Thus the interaction of two D-branes is viewed as an exchange of closed string states, therefore it is computed with a tree level diagram in which two boundary states are connected by means of a closed string propagator.

By introducing back-ground fields $B_{\mu\nu}$ and A_α a $U(1)$ gauge field (which lives in the D-brane) in the string σ-model one obtains mixed boundary conditions for string, these fields appear in the boundary states and modify the tensions of the branes. Mixed boundary state formalism enables us to consider the problems not easily accessible to the canonical approach via open strings.

Mixed boundary conditions have been used for studying properties of D-branes in back-ground fields [8, 9, 10, 11, 12, 13, 14, 15, 16]. In Ref.[15],the interaction between D_0 and D_6 branes with back-ground gauge fields has been discussed. In Ref.[16] we applied mixed boundary conditions for the closed bosonic string and studied the interactions of the branes in spacetime with compactification on tori. Inclusion of fermionic degrees of freedom is non trivial and requires its own techniques. This is what we take on in this article.

We use the covariant formalism to extract the boundary states which now involve apart from the bosonic and fermionic components, due to the covariance the ghosts and superghosts elements. Then we compute the interaction amplitude between two mixed branes with arbitrary dimensions p_1 and p_2 and field strengths $\mathcal{F}_1$ and $\mathcal{F}_2$ as a closed string tree level diagram. Then we proceed to study the above considerations when certain directions are compactified. Finally to elucidate our general computations we apply our results to special cases. It is worth emphasizing that part of these special cases are either inaccessible to the canonical methods and the other part are very difficult to handle by canonical formulation. Among the special cases that will be considered is the parallel m_{p_1} and m_{p_2}-branes with the same total field strength. The $NS \otimes NS$ sector interaction for $p_2 - p_1 = 4$ vanishes, also for $p_1 = p_2$ the total interaction vanishes. Other examples include different internal fields are : parallel $m_1 - m_{1'}$, perpendicular $m_1 - m_{1'}$, $m_2 - m_0$, parallel $m_2 - m_{2'}$, perpendicular $m_2 - m_{2'}$ and parallel $m_5 - m_1$ branes. They are considered to clear more properties of the field strengths and compactification effects on interaction amplitude. Finally contribution of the massless states on the amplitude for the NS-NS and R-R sectors separately will be obtained. In this article we denote a mixed brane with dimension "p" by notation "m_p-brane".

2 Boundary state

First we develop the boundary state formalism for the branes with background gauge fields. Deriving the boundary conditions from a σ-model action, we turn them in to boundary state equations which we will solve in the next subsection.

2.1 Boundary state equations

A σ-model action with $B_{\mu\nu}$ field and two boundary terms [17] corresponding to the two m_{p_1} and m_{p_2}-branes gauge fields is

$$S = -\frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma\Big(\sqrt{-g}g^{ab}G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + \epsilon^{ab}B_{\mu\nu}\partial_a X^\mu \partial_b X^\nu\Big)$$
$$-\frac{1}{2\pi\alpha'}\int_{(\partial\Sigma)_1} d\sigma A^{(1)}_{\alpha_1}\partial_\sigma X^{\alpha_1} + \frac{1}{2\pi\alpha'}\int_{(\partial\Sigma)_2} d\sigma A^{(2)}_{\alpha_2}\partial_\sigma X^{\alpha_2} \quad , \tag{1}$$

where Σ is the world sheet of closed string exchanged between the branes and $(\partial\Sigma)_1$ and $(\partial\Sigma)_2$ are two boundaries of the world sheet. The first boundary is at $\tau = 0$ and the second at $\tau = \tau_0$. The two $U(1)$ gauge fields $A^{(1)}_{\alpha_1}$ and $A^{(2)}_{\alpha_2}$ live in m_{p_1} and m_{p_2}-branes respectively. $\alpha_1, \beta_1 \in \{0, \bar{\alpha}_1, ..., \bar{\alpha}_{p_1}\}$, this set shows directions along the m_{p_1}-brane and $\{i_1\}$ show directions perpendicular to it. Likely $\alpha_2, \beta_2 \in \{0, \bar{\beta}_1, ..., \bar{\beta}_{p_2}\}$ and $\{i_2\}$ for m_{p_2}-brane. $G_{\mu\nu}$ and $B_{\mu\nu}$ are usual back-ground fields. Let $B_{\mu\nu}(X)$ and $G_{\mu\nu}(X)$ be constant fields. Variation of this action with respect to $X^\mu(\sigma,\tau)$ gives the boundary state equations and equation of motion of $X^\mu(\sigma,\tau)$. Using the convention $\epsilon^{01} = -\epsilon^{10} = 1$, we obtain

$$\Big(\partial_\tau X^{\alpha_1} + \mathcal{F}^{\alpha_1}_{(1)\ \beta_1}\partial_\sigma X^{\beta_1} - B^{\alpha_1}{}_{j_1}\partial_\sigma X^{j_1}\Big)_{\tau=0} \mid B^1_x\rangle = 0 \quad ,$$
$$(\delta X^{i_1})_{\tau=0} \mid B^1_x\rangle = 0 \quad ,$$
$$\Big(\partial_\tau X^{\alpha_2} + \mathcal{F}^{\alpha_2}_{(2)\ \beta_2}\partial_\sigma X^{\beta_2} - B^{\alpha_2}{}_{j_2}\partial_\sigma X^{j_2}\Big)_{\tau_0} \mid B^2_x\rangle = 0 \quad ,$$
$$(\delta X^{i_2})_{\tau_0} \mid B^2_x\rangle = 0 \quad , \tag{2}$$

where

$$\mathcal{F}_{(1)\alpha_1\beta_1} \equiv \partial_{\alpha_1}A^{(1)}_{\beta_1} - \partial_{\beta_1}A^{(1)}_{\alpha_1} - B_{\alpha_1\beta_1} \quad ,$$
$$\mathcal{F}_{(2)\alpha_2\beta_2} \equiv \partial_{\alpha_2}A^{(2)}_{\beta_2} - \partial_{\beta_2}A^{(2)}_{\alpha_2} - B_{\alpha_2\beta_2} \quad . \tag{3}$$

The transverse coordinates of the two branes $\{y_1^{i_1}\}$ and $\{y_2^{i_2}\}$ are kept fixed *i.e*

$$[X^{i_1}(\sigma,\tau) - y_1^{i_1}]_{\tau=0} \mid B^1_x\rangle = 0 \quad ,$$
$$[X^{i_2}(\sigma,\tau) - y_2^{i_2}]_{\tau_0} \mid B^2_x\rangle = 0 \quad . \tag{4}$$

These imply $\partial_\sigma X^{j_1}$ (, $\partial_\sigma X^{j_2}$) vanish and be dropped from the first (third) equation of (2).

Solution of the equations of motion of the closed string is

$$X^\mu(\sigma,\tau) = x^\mu + 2\alpha' p^\mu \tau + 2L^\mu \sigma + \frac{i}{2}\sqrt{2\alpha'}\sum_{m\neq 0}\frac{1}{m}\Big(\alpha^\mu_m e^{-2im(\tau-\sigma)} + \tilde{\alpha}^\mu_m e^{-2im(\tau+\sigma)}\Big) \ , \tag{5}$$

where L^μ is zero for non compact directions. For compact directions we have $L^\mu = N^\mu R^\mu$ and $p^\mu = \frac{M^\mu}{R^\mu}$, in which N^μ is the winding number and M^μ is the momentum number of closed string state, and R_μ is the radius of compactification of X^μ-direction. Combining the solution of the equation of motion and the boundary state equations we obtain

$$\Big(p^{\alpha_2} + \frac{1}{\alpha'}\mathcal{F}^{\alpha_2}_{(2)\ \ \beta_2} L^{\beta_2}\Big) \mid B^2_x, \tau_0\rangle = 0 \ \ , \tag{6}$$

$$\Big((1-\mathcal{F}_2)^{\alpha_2}{}_{\beta_2}\ \alpha^{\beta_2}_n e^{-2in\tau_0} + (1+\mathcal{F}_2)^{\alpha_2}{}_{\beta_2}\ \tilde{\alpha}^{\beta_2}_{-n} e^{2in\tau_0}\Big) \mid B^2_x, \tau_0\rangle = 0 \ \ , \tag{7}$$

$$\Big(\alpha^{i_2}_n e^{-2in\tau_0} - \tilde{\alpha}^{i_2}_{-n} e^{2in\tau_0}\Big) \mid B^2_x, \tau_0\rangle = 0 \ \ , \tag{8}$$

$$(x^{i_2} + 2\alpha' p^{i_2}\tau_0 - y^{i_2}_2) \mid B^2_x, \tau_0\rangle = 0 \ \ , \tag{9}$$

$$L^{i_2} \mid B^2_x, \tau_0\rangle = 0 \ \ , \tag{10}$$

The boundary conditions on the fermionic degrees of freedom should be imposed on both $R\otimes R$ and $NS\otimes NS$ sectors. World sheet supersymmetry requires the two sectors to satisfy the boundary conditions,

$$\Big[(\psi^{\alpha_2} - i\eta_2\tilde{\psi}^{\alpha_2}) - \mathcal{F}^{\alpha_2}_{(2)\ \ \beta_2}\ (\psi^{\beta_2} + i\eta_2\tilde{\psi}^{\beta_2})\Big]_{\tau_0} \mid B^2_\psi, \eta_2, \tau_0\rangle = 0 \ \ , \tag{11}$$

$$(\psi^{i_2} + i\eta_2\tilde{\psi}^{i_2})_{\tau_0} \mid B^2_\psi, \eta_2, \tau_0\rangle = 0 \ \ , \tag{12}$$

where $\eta_2 = \pm 1$ is the phase used to make GSO projection easily. These states preserve half of the world sheet supersymmetry. Expanding the fermions in Fourier modes the boundary conditions become

$$\Big(\psi^{i_2}_k + i\eta_2 e^{4ik\tau_0}\tilde{\psi}^{i_2}_{-k}\Big) \mid B^2_\psi, \eta_2, \tau_0\rangle = 0 \ \ , \tag{13}$$

$$\Big(\psi^{\alpha_2}_k - i\eta_2 Q^{\alpha_2}_{(2)\ \ \beta_2}\ e^{4ik\tau_0}\tilde{\psi}^{\beta_2}_{-k}\Big) \mid B^2_\psi, \eta_2, \tau_0\rangle = 0 \ \ , \tag{14}$$

where the index k is integer in the R-R sector and half-integer in the NS-NS sector. The matrix Q_2 is

$$Q_2 \equiv (1 - \mathcal{F}_2)^{-1}(1 + \mathcal{F}_2) \ , \tag{15}$$

since $\mathcal{F}_2$ is antisymmetric , Q_2 is an orthogonal matrix. Since we use the covariant formalism we shall introduce ghost for the bosonic and fermionic gauge (reparametrization) degrees of freedom. Let the ghost coordinates be $b(\sigma, \tau)$ and $c(\sigma, \tau)$. Vanishing of the variation of the ghosts action gives the ghosts boundary conditions. Therefore the ghost modes satisfy the following boundary conditions

$$\left(b_n \ e^{-2in\tau_0} - \tilde{b}_{-n} \ e^{2in\tau_0}\right) \mid B^2_{gh}, \tau_0\rangle = 0 \ \ , \tag{16}$$

$$\left(c_n \ e^{-2in\tau_0} + \tilde{c}_{-n} \ e^{2in\tau_0}\right) \mid B^2_{gh}, \tau_0\rangle = 0 \ \ , \tag{17}$$

where n is non zero integer. The same consideration also determine the boundary conditions for superghost coordinates β,$\tilde{\beta}$,γ, $\tilde{\gamma}$

$$\left(\gamma_k + i\eta_2 \tilde{\gamma}_{-k} \ e^{4ik\tau_0}\right) \mid B^2_{sgh}, \eta_2, \tau_0\rangle = 0 \ \ , \tag{18}$$

$$\left(\beta_k + i\eta_2 \tilde{\beta}_{-k} \ e^{4ik\tau_0}\right) \mid B^2_{sgh}, \eta_2, \tau_0\rangle = 0 \ \ . \tag{19}$$

The index k as previous is integer in the R-R sector and half-integer in the NS-NS sector.

2.2 Solutions of boundary state equations

To find the boundary states we shall proceed to solve the equations 6-10, 13-14 and 16-19. Equations (6-10) have the solution,

$$\mid B^2_x, \tau_0\rangle = \sum_{\{p^{\alpha_2}\}} \mid B^2_x, \tau_0, p^0, p^{\bar{\beta}_1}, ..., p^{\bar{\beta}_{p_2}}\rangle \ \ , \tag{20}$$

$$\begin{aligned} \mid B^2_x, \tau_0, p^0, p^{\bar{\beta}_1}, ..., p^{\bar{\beta}_{p_2}}\rangle &= \frac{T_{p_2}}{2}\sqrt{\det(1-\mathcal{F}_2)} \ e^{i\alpha'\tau_0 \sum_{i_2}(p^{i_2}_{op})^2} \delta^{(d-p_2-1)}(x^{i_2} - y^{i_2}_2) \\ &\times \exp\left[-\sum_{m=1}^{\infty} \frac{1}{m} e^{4im\tau_0} \alpha^{\mu}_{-m} S^{(2)}_{\mu\nu} \tilde{\alpha}^{\nu}_{-m}\right] \mid 0\rangle \prod_{i_2} \mid p^{i_2}_L = p^{i_2}_R = 0\rangle \prod_{\alpha_2} \mid p^{\alpha_2}\rangle \ \ , \end{aligned} \tag{21}$$

where the matrix $S^{\mu}_{(2)\ \nu}$ is

$$S^{\mu}_{(2)\ \nu} = (Q^{\alpha_2}_{(2)\ \beta_2} \ , \ -\delta^{i_2}{}_{j_2}) \ \ , \tag{22}$$

and T_{p_2} is a constant depending on the tension of m_{p_2}-brane [3, 6]. The overall factor $\sqrt{\det(1-\mathcal{F}_2)}$ is expected by the path integral with boundary action [13, 18]. In (20) the summation over $\{p^{\alpha_2}\}$ can change to a sum over winding numbers $\{N^{\alpha_{2c}}\}$ due to the equation (6) which implies

$$p^{\alpha_2} = -\frac{1}{\alpha'}\sum_{\beta_{2c}}\mathcal{F}^{\alpha_2}_{(2)\ \beta_{2c}}\ \ell^{\beta_{2c}} \quad , \tag{23}$$

where $\ell^{\beta_{2c}} = N^{\beta_{2c}}R^{\beta_{2c}}$ and β_{2c} shows the direction along m_{p_2}-brane which is compact. This relation implies that the closed string state can have non zero momentum along the world brane if there are non zero back-ground internal gauge fields and at least one of the brane directions is compact. This relation correlates the momentum of closed string state along the brane directions to its winding numbers. For compact directions of the brane, the closed string state also has momentum numbers $\{M^{\alpha_{2c}}\}$, therefore when α_2 in (23) refers to the compact directions of brane, we have

$$\frac{M^{\alpha_{2c}}}{R^{\alpha_{2c}}} = -\frac{1}{\alpha'}\sum_{\beta_{2c}}\mathcal{F}^{\alpha_{2c}}_{(2)\ \beta_{2c}}R^{\beta_{2c}}N^{\beta_{2c}} \quad , \tag{24}$$

this is a relation between momentum numbers and winding numbers of a given closed string state, more details can be found in [16] where the pure bosonic case is discussed.

Ghost part of boundary state has the form

$$|\ B^2_{gh},\tau_0\rangle = \exp\left[\sum_{m=1}^{\infty} e^{4im\tau_0}(c_{-m}\tilde{b}_{-m} - b_{-m}\tilde{c}_{-m})\right]\frac{c_0+\tilde{c}_0}{2}\ |\ q=1\rangle\ |\ \tilde{q}=1\rangle \quad , \tag{25}$$

Let us denote the fermionic modes in the R-R sector with d^μ_n and in the NS-NS sector with b^μ_r, therefore the fermionic and the superghost parts of the NS-NS sector boundary state in the $(-1,-1)$ picture is

$$|\ B^2_\psi,\eta_2,\tau_0\rangle_{NS} = \exp\left[i\eta_2\sum_{r=1/2}^{\infty} e^{4ir\tau_0}b^\mu_{-r}S^{(2)}_{\mu\nu}\tilde{b}^\nu_{-r}\right]|\ 0\rangle \quad , \tag{26}$$

$$|\ B^2_{sgh},\eta_2,\tau_0\rangle_{NS} = \exp\left[i\eta_2\sum_{r=1/2}^{\infty} e^{4ir\tau_0}(\gamma_{-r}\tilde{\beta}_{-r} - \beta_{-r}\tilde{\gamma}_{-r})\right]|\ P=-1,\tilde{P}=-1\rangle \quad . \tag{27}$$

The fermionic and the superghost parts of the R-R sector boundary state in the $(-1/2,-3/2)$ picture is

$$|\ B^2_\psi,\eta_2,\tau_0\rangle_R = \frac{1}{\sqrt{\det(1-\mathcal{F}_2)}}\ \exp\left[i\eta_2\sum_{m=1}^{\infty} e^{4im\tau_0}d^\mu_{-m}S^{(2)}_{\mu\nu}\tilde{d}^\nu_{-m}\right]|\ B^2_\psi,\eta_2\rangle^{(0)}_R \quad , \tag{28}$$

$$| B^2_{sgh}, \eta_2, \tau_0\rangle_R = \exp\Big[i\eta_2 \sum_{m=1}^{\infty} e^{4im\tau_0}(\gamma_{-m}\tilde{\beta}_{-m} - \beta_{-m}\tilde{\gamma}_{-m}) + i\eta_2\gamma_0\tilde{\beta}_0\Big] \mid P = -1/2, \tilde{P} = -3/2\rangle \quad ,(29)$$

where the superghost vacuum is in the $(-1/2, -3/2)$ picture and is annihilated by β_0 and $\tilde{\gamma}_0$ [19] and $| B^2_\psi, \eta_2\rangle^{(0)}_R$ is the fermionic zero mode boundary state. Appearance of the determinant in the denominator is the consequence of the path integral over the fermions with fermionic boundary term. Comparison of (21) and (28) implies that in the R-R sector the normalizing determinant factors of the bosonic boundary determinant and its fermionic partner cancel. However this factor remains in the NS-NS sector.

We now derive the explicit form of $| B^2_\psi, \eta_2\rangle^{(0)}_R$ both in type IIA and type IIB theories. It obeys the equations (13) and (14) with $k = 0$, *i.e.*

$$(d^{i_2}_0 + i\eta_2\tilde{d}^{i_2}_0) \mid B^2_\psi, \eta_2\rangle^{(0)}_R = 0 \quad , \tag{30}$$

$$(d^{\alpha_2}_0 - i\eta_2 Q^{\alpha_2}_{(2)\ \beta_2}\tilde{d}^{\beta_2}_0) \mid B^2_\psi, \eta_2\rangle^{(0)}_R = 0 \quad , \tag{31}$$

or in combined form,

$$(d^\mu_0 - i\eta_2 S^\mu_{(2)\ \nu}\tilde{d}^\nu_0) \mid B^2_\psi, \eta_2\rangle^{(0)}_R = 0 \quad . \tag{32}$$

The vacuum for the fermionic zero modes d^μ_0 and $\tilde{d}^\mu_0$ can be written as [6]

$$| A\rangle \mid \tilde{B}\rangle = \lim_{z,\bar{z}\to 0} S^A(z)\tilde{S}^B(\bar{z}) \mid 0\rangle \quad , \tag{33}$$

where S^A and $\tilde{S}^B$ are the spin fields in the 32-dimensional Majorana representation. We use a chiral representation for the 32×32 Γ-matrices of SO(1,9) as in reference [6]. Also the action of the Ramond oscillators d^μ_n and $\tilde{d}^\mu_n$ on the state $| A\rangle \mid \tilde{B}\rangle$ are given in [6], therefore we consider solution of (32) of the form (like [7]),

$$| B^2_\psi, \eta_2\rangle^{(0)}_R = \mathcal{M}^{(\eta_2)}_{AB} \mid A\rangle \mid \tilde{B}\rangle \quad , \tag{34}$$

therefore the 32×32 matrix $\mathcal{M}^{(\eta_2)}$ satisfies the following equation

$$(\Gamma^\mu)^T\mathcal{M}^{(\eta_2)} - i\eta_2 S^\mu_{(2)\ \nu}\Gamma_{11}\mathcal{M}^{(\eta_2)}\Gamma^\nu = 0 \quad . \tag{35}$$

For this equation we consider a solution with the form

$$\mathcal{M}^{(\eta_2)} = C\Gamma^0\Gamma^{\bar{\beta}_1}...\Gamma^{\bar{\beta}_{p_2}}\Big(\frac{1 + i\eta_2\Gamma_{11}}{1 + i\eta_2}\Big)G_2 \quad , \tag{36}$$

where C is the charge conjugation matrix and $\bar{\beta}_i$'s show the space directions of the m_{p_2}-brane world volume. For the case of $\mathcal{F}_2 = 0$, G_2 must be equal to the unit matrix, in this case (36) reduces to the equation (2.22) of [7]. From (35) and (36) we see that G_2 must satisfy the equation

$$\Gamma^\alpha G_2 = Q^\alpha_{(2)\ \beta}\, G_2 \Gamma^\beta \quad , \quad \alpha, \beta \in \{0, \bar{\beta}_1, ..., \bar{\beta}_{p_2}\} \ . \tag{37}$$

Therefore matrix G_2 has the solution with the conventional form

$$G_2 = e^{\frac{1}{2}(\mathcal{F}_2)_{\alpha\beta}\Gamma^\alpha\Gamma^\beta} \quad , \tag{38}$$

Indeed one must expand the exponential with the convention that all gamma matrices anti commute, therefore there are a finite number of terms. This convention is in Ref.[11, 13]. For example for m_p-brane with $p = 1$ along X^1, $p = 2$ along (X^1, X^2) and $p = 3$ along (X^1, X^2, X^3) directions, respectively we have

$$G_2 = 1 + \mathcal{F}_{(2)01}\Gamma^0\Gamma^1 \quad , \tag{39}$$

$$G_2 = 1 + \mathcal{F}_{(2)01}\Gamma^0\Gamma^1 + \mathcal{F}_{(2)02}\Gamma^0\Gamma^2 + \mathcal{F}_{(2)12}\Gamma^1\Gamma^2 \quad , \tag{40}$$

$$G_2 = 1 + \frac{1}{2}\mathcal{F}_{(2)\alpha\beta}\Gamma^\alpha\Gamma^\beta + (\mathcal{F}_{(2)01}\mathcal{F}_{(2)23} - \mathcal{F}_{(2)02}\mathcal{F}_{(2)13} + \mathcal{F}_{(2)03}\mathcal{F}_{(2)12})\Gamma^0\Gamma^1\Gamma^2\Gamma^3 \quad , \tag{41}$$

and $\alpha, \beta = 0, 1, 2, 3$.

This special representation of Γ-matrices allow us to decompose the spinors in chiral and anti-chiral components ($A = (\alpha, \dot{\alpha})$) with sixteen dimensional indices α and $\dot{\alpha}$. In the type IIA theory p_2 is even, therefore $\mathcal{M}^{(\eta_2)}$ is a block-diagonal matrix, whereas in the type IIB theory p_2 is odd and therefore $\mathcal{M}^{(\eta_2)}$ is in the form of an off diagonal matrix with matrices as its elements. Thus in the sixteen-dimensional notation, $\mid B^2_\psi, \eta_2\rangle^{(0)}_R$ becomes

$$\mid B^2_\psi, \eta_2\rangle^{(0)}_R = M_{\alpha\beta} \mid \alpha\rangle_{-1/2} \mid \tilde{\beta}\rangle_{-3/2} - i\eta_2 M_{\dot{\alpha}\dot{\beta}} \mid \dot{\alpha}\rangle_{-1/2} \mid \tilde{\dot{\beta}}\rangle_{-3/2} \quad for IIA \ , \tag{42}$$

$$\mid B^2_\psi, \eta_2\rangle^{(0)}_R = M_{\dot{\alpha}\beta} \mid \dot{\alpha}\rangle_{-1/2} \mid \tilde{\beta}\rangle_{-3/2} - i\eta_2 M_{\alpha\dot{\beta}} \mid \alpha\rangle_{-1/2} \mid \tilde{\dot{\beta}}\rangle_{-3/2} \quad for IIB \ , \tag{43}$$

where the matrix M_{AB} has definition

$$M_{AB} \equiv \begin{pmatrix} M_{\alpha\beta} & M_{\alpha\dot{\beta}} \\ M_{\dot{\alpha}\beta} & M_{\dot{\alpha}\dot{\beta}} \end{pmatrix} = (C\Gamma^0\Gamma^{\bar{\beta}_1}...\Gamma^{\bar{\beta}_{p_2}}G_2)_{AB} \ . \tag{44}$$

2.3 GSO projection of the boundary state

For both NS-NS and R-R sectors the complete boundary state can be written as the following product

$$\mid B_2, \eta_2, \tau_0\rangle_{R,NS} = \mid B_x^2, \tau_0\rangle \mid B_{gh}^2, \tau_0\rangle \mid B_\psi^2, \eta_2, \tau_0\rangle_{R,NS} \mid B_{sgh}^2, \eta_2, \tau_0\rangle_{R,NS} \quad . \tag{45}$$

The projected boundary state in the NS-NS sector is [7]

$$\mid B_2, \tau_0\rangle_{NS} = \frac{1-(-1)^{F+G}}{2}\frac{1-(-1)^{\tilde{F}+\tilde{G}}}{2} \mid B_2, +, \tau_0\rangle_{NS} \quad , \tag{46}$$

where F and G are

$$F = \sum_{r=1/2}^{\infty} b^\mu_{-r} b_{r\mu} \qquad , \quad G = -\sum_{r=1/2}^{\infty}(\gamma_{-r}\beta_r + \beta_{-r}\gamma_r) \quad . \tag{47}$$

Similar definitions hold for $\tilde{F}$ and $\tilde{G}$. Therefore projected state becomes

$$\mid B_2, \tau_0\rangle_{NS} = \frac{1}{2}(\mid B_2, +, \tau_0\rangle_{NS} - \mid B_2, -, \tau_0\rangle_{NS}\ ) \quad . \tag{48}$$

In the R-R sector the projection is

$$\mid B_2, \tau_0\rangle_{R} = \frac{1+(-1)^p(-1)^{F+G}}{2}\frac{1-(-1)^{\tilde{F}+\tilde{G}}}{2} \mid B_2, +, \tau_0\rangle_{R} \quad , \tag{49}$$

where p is even for type IIA and odd for type IIB, and

$$(-1)^F = \Gamma_{11}(-1)^{\sum_{m=1}^{\infty} d^\mu_{-m} d_{m\mu}} \quad , \quad G = -\gamma_0\beta_0 - \sum_{m=1}^{\infty}(\gamma_{-m}\beta_m + \beta_{-m}\gamma_m) \quad . \tag{50}$$

Finally the projected state is

$$\mid B_2, \tau_0\rangle_{R} = \frac{1}{2}(\mid B_2, +, \tau_0\rangle_{R} + \mid B_2, -, \tau_0\rangle_{R}\ ) \quad . \tag{51}$$

Equation (48) and (51) are similar to the case in which $\mathcal{F}_2 = 0$.

3 Mixed branes interaction

Before calculation of the interaction amplitude, let us introduce some notations for the positions of these two mixed branes. The set $\{i\}$ shows indices for directions perpendicular to the both of the branes, $\{u\}$ for the directions along the both of them, $\{\alpha'_1\}$ for directions along m_{p_1} and perpendicular to the m_{p_2} and $\{\alpha'_2\}$ for directions along m_{p_2} and perpendicular to the m_{p_1} -branes. It can be seen that for example $\{i_1\} = \{i\}\bigcup\{\alpha'_2\}$, $\{\alpha_1\} = \{u\}\bigcup\{\alpha'_1\}$.

3.1 The amplitude for the NS-NS sector

For calculation of the amplitude we need to the conjugate form of the boundary states. In the NS-NS sector there are

$$ {}_{NS}\langle B^1_\psi, \eta_1 \mid = \langle 0 \mid e^{-i\eta_1 \sum_{r=1/2}^{\infty} b^\mu_r S^{(1)}_{\mu\nu} \tilde{b}^\nu_r} \ , \tag{52} $$

$$ {}_{NS}\langle B^1_{sgh}, \eta_1 \mid = \langle P=-1, \tilde{P}=-1 \mid e^{-i\eta_1 \sum_{r=1/2}^{\infty} (\tilde{\beta}_r \gamma_r - \tilde{\gamma}_r \beta_r)} \ . \tag{53} $$

The two mixed branes simply interact via exchange of closed strings, and the amplitude is

$$ \mathcal{A} = \langle B_1 \mid D \mid B_2, \tau_0 = 0 \rangle \ , \tag{54} $$

where "D" is closed string propagator and one must use the GSO projected boundary states. The NS-NS sector amplitude becomes

$$ \begin{aligned} \mathcal{A}_{NS-NS} \ = \ & \frac{T_{p_1} T_{p_2}}{8(2\pi)^{d_i}} \alpha' \sqrt{\det(1-\mathcal{F}_1)\det(1-\mathcal{F}_2)} \int_0^\infty dt \Big\{ \Big(\sqrt{\frac{\pi}{\alpha' t}} \Big)^{d_{i_n}} \\ & \times e^{-\frac{1}{4\alpha' t} \sum_{i_n} (y_1^{i_n} - y_2^{i_n})^2} \prod_{i_c} \Theta_3 \Big(\frac{y_1^{i_c} - y_2^{i_c}}{2\pi R_{i_c}} \mid \frac{i\alpha' t}{\pi (R_{i_c})^2} \Big) \\ & \times \frac{1}{q} \Big(\prod_{n=1}^{\infty} \Big[\Big(\frac{1-q^{2n}}{1+q^{2n-1}} \Big)^2 \frac{\det(1+S_1 S_2^T q^{2n-1})}{\det(1-S_1 S_2^T q^{2n})} \Big] \\ & - \prod_{n=1}^{\infty} \Big[\Big(\frac{1-q^{2n}}{1-q^{2n-1}} \Big)^2 \frac{\det(1-S_1 S_2^T q^{2n-1})}{\det(1-S_1 S_2^T q^{2n})} \Big] \Big) \\ & \times \sum_{\{N^{u_c}\}} \Big[(2\pi)^{d_u} \prod_u [\delta(p_1^u - p_2^u)] \ \exp[\frac{i}{\alpha'} \ell^{u_c} (\mathcal{F}_{(1)}^{\alpha'_1}{}_{u_c} y_2^{\alpha'_1} - \mathcal{F}_{(2)}^{\alpha'_2}{}_{u_c} y_1^{\alpha'_2})] \\ & \times \exp[-\frac{t}{\alpha'} \ell^{u_c} \ell^{v_c} (\eta_{u_c v_c} + \mathcal{F}^u_{(1)\ u_c} \mathcal{F}_{(2)\ u v_c} + \mathcal{F}^{\alpha'_1}_{(1)\ u_c} \mathcal{F}^{\alpha'_1}_{(1)\ v_c} + \mathcal{F}^{\alpha'_2}_{(2)\ u_c} \mathcal{F}^{\alpha'_2}_{(2)\ v_c})] \Big] \Big\} \end{aligned} \tag{55} $$

where $q = e^{-2t}$. In this formula $p_1^u = -\frac{1}{\alpha'} \mathcal{F}^u_{(1)\ v_c} N^{v_c} R^{v_c}$ and $p_2^u = -\frac{1}{\alpha'} \mathcal{F}^u_{(2)\ v_c} N^{v_c} R^{v_c}$. Indices $\{u_c, v_c, ...\}$ show compact part of $\{u\}$, d_u and d_i are dimensions of $\{X^u\}$ and $\{X^i\}$ respectively. Also $\{i_n\}$ and $\{i_c\}$ are non compact part and compact part of $\{i\}$ region respectively. ℓ^{u_c} as previous is $N^{u_c} R^{u_c}$. Note that determinant in the denominators comes from the world sheet bosons and in the numerators from the fermions. This amplitude is symmetric under the exchange of the indices "1" and "2", *i.e* $\mathcal{A}_{NS}(1,2) = \mathcal{A}^*_{NS}(2,1)$ as expected. In this amplitude we see how the effects of compactification appear. Later we will see that this compactification structure will be repeated in the R-R sector.

The momentum delta functions put severe restrictions on the summation. The term corresponding to $N^{u_c} = 0$ for all u_c, gives $p_1^u = p_2^u = 0$ and is always present. Other terms occur only if the two internal back-ground fields and radii of compactification with some sets $\{N^{u_c}\}$ satisfy the relation $\sum_{v_c}(\mathcal{F}^{u}_{(1)\ v_c}N^{v_c}R^{v_c}) = \sum_{v_c}(\mathcal{F}^{u}_{(2)\ v_c}N^{v_c}R^{v_c})$ for all u.

Now suppose there is no compact direction, then (55) simplifies,

$$\begin{aligned}\mathcal{A}^{(nc)}_{NS-NS} \;=\; & \frac{T_{p_1}T_{p_2}}{8(2\pi)^{d_i}}\alpha' V_u\sqrt{\det(1-\mathcal{F}_1)\det(1-\mathcal{F}_2)}\int_0^\infty dt\Big\{\Big(\sqrt{\frac{\pi}{\alpha' t}}\Big)^{d_i} e^{-\frac{1}{4\alpha' t}\sum_i (y_1^i-y_2^i)^2} \\ & \times\frac{1}{q}\Big(\prod_{n=1}^{\infty}\Big[\Big(\frac{1-q^{2n}}{1+q^{2n-1}}\Big)^2\frac{\det(1+S_1S_2^Tq^{2n-1})}{\det(1-S_1S_2^Tq^{2n})}\Big] \\ & -\prod_{n=1}^{\infty}\Big[\Big(\frac{1-q^{2n}}{1-q^{2n-1}}\Big)^2\frac{\det(1-S_1S_2^Tq^{2n-1})}{\det(1-S_1S_2^Tq^{2n})}\Big]\Big)\Big\}\ ,\end{aligned} \tag{56}$$

where V_u is the common world volume of the two mixed branes.

3.2 The R-R sector amplitude

In the R-R sector there are

$$_R\langle B^1_\psi,\eta_1 \mid\ = \langle A \mid \langle \tilde{B} \mid \mathcal{N}^{(\eta_1)}_{AB}e^{-i\eta_1\sum_{m=1}^{\infty}d^\mu_m S^{(1)}_{\mu\nu}\tilde{d}^\nu_m}\ , \tag{57}$$

where $\mathcal{N}^{(\eta_1)}$ is given by

$$\mathcal{N}^{(\eta_1)} = (-1)^{p_1}C\Gamma^0\Gamma^{\bar{\alpha}_1}...\Gamma^{\bar{\alpha}_{p_1}}G_1\Big(\frac{1-i\eta_1\Gamma_{11}}{1+i\eta_1}\Big)\ , \tag{58}$$

and

$$_R\langle B^1_{sgh},\eta_1 \mid\ = \langle P=-3/2,\tilde{P}=-1/2 \mid e^{i\eta_1\beta_0\tilde{\gamma}_0 - i\eta_1\sum_{m=1}^{\infty}(\gamma_m\tilde{\beta}_m-\beta_m\tilde{\gamma}_m)}\ . \tag{59}$$

In calculation of $\mathcal{A}_{R-R}(\eta_1,\eta_2) = {}_R\langle B_1,\eta_1 \mid D \mid B_2,\eta_2\rangle_R$ we see that zero mode contribution of the superghost is

$${}^{(0)}_{R}\langle B^1_{sgh},\eta_1 \mid B^2_{sgh},\eta_2\rangle^{(0)}_R = \sum_{m=0}^{\infty}(\eta_1\eta_2)^m\ . \tag{60}$$

which for $\eta_1\eta_2 = +1$ is divergent, and for $\eta_1\eta_2 = -1$ is an alternating sum. This expression needs to be regularized to have a meaning. We introduce a special regularization scheme similar Ref.[19]. For this we define

$${}^{(0)}_{R}\langle B_1,\eta_1 \mid B_2,\eta_2\rangle^{(0)}_R \equiv \lim_{x\to 1}{}^{(0)}_{R}\langle B^1_{sgh},\eta_1 \mid x^{2G_0} \mid B^2_{sgh},\eta_2\rangle^{(0)}_R\ {}^{(0)}_{R}\langle B^1_\psi,\eta_1 \mid B^2_\psi,\eta_2\rangle^{(0)}_R\ , \tag{61}$$

similar to the equation (3.8) of Ref. [7]. Also G_0 is defined in (50), i.e. $G_0 = -\gamma_0\beta_0$, therefore

$${}^{(0)}_{R}\langle B^1_{sgh},\eta_1 \mid x^{2G_0} \mid B^2_{sgh},\eta_2\rangle^{(0)}_R = \frac{1}{1-\eta_1\eta_2x^2}\ . \tag{62}$$

For alternating sum (i.e. $\eta_1\eta_2 = -1$) this becomes $\frac{1}{1+x^2}$, and for $x = 1$ reduces to $\frac{1}{2}$. For $\eta_1\eta_2 = +1$ (i.e. $\eta_1 = \eta_2 \equiv \eta$) is

$$
{}_R^{(0)}\langle B^1_{sgh}, \eta \mid x^{2G_0} \mid B^2_{sgh}, \eta\rangle_R^{(0)} = \frac{1}{1-x^2} \quad . \tag{63}
$$

By an appropriate insertion of $\beta_0, \gamma_0, \tilde{\beta}_0$ and $\tilde{\gamma}_0$ in the left hand side of (63), projecting it out, therefore

$$
\lim_{x\to 1} {}_R^{(0)}\langle B^1_{sgh}, \eta \mid x^{-2\gamma_0\beta_0}\delta(\beta_0 - \frac{1}{4\pi}\gamma_0)\delta(\tilde{\beta}_0 + \frac{1}{4\pi}\tilde{\gamma}_0) \mid B^2_{sgh}, \eta\rangle_R^{(0)} = 1 \quad . \tag{64}
$$

This gives a modified partition function for $\eta_1 = \eta_2$, which is regular. Also zero mode part of the fermions becomes

$$
\mathcal{A}^{0(R)}_{\psi}(\eta_1, \eta_2) \quad \equiv \quad {}_R^{(0)}\langle B^1_{\psi}, \eta_1 \mid B^2_{\psi}, \eta_2\rangle_R^{(0)} = \mathrm{Tr}\Big(\mathcal{M}^{(\eta_2)}C^{-1}\mathcal{N}^{(\eta_1)^T}C^{-1}\Big) \quad , \tag{65}
$$

this gives

$$
\mathcal{A}^{0(R)}_{\psi}(+,-) = \mathcal{A}^{0(R)}_{\psi}(-,+) = 2\zeta \quad , \tag{66}
$$

$$
\mathcal{A}^{0(R)}_{\psi}(+,+) = \mathcal{A}^{0(R)}_{\psi}(-,-) = \zeta' \quad , \tag{67}
$$

where ζ and ζ' have definition as

$$
\zeta \equiv -\frac{1}{2}\mathrm{Tr}\Big[G_1 C^{-1} G_2^T C(\Gamma^{\bar{\beta}_{p_2}}...\Gamma^{\bar{\beta}_1})(\Gamma^{\bar{\alpha}_1}...\Gamma^{\bar{\alpha}_{p_1}})\Big] \quad , \tag{68}
$$

$$
\zeta' \equiv i\mathrm{Tr}\Big[G_1 C^{-1} G_2^T C(\Gamma^{\bar{\beta}_{p_2}}...\Gamma^{\bar{\beta}_1})(\Gamma^{\bar{\alpha}_1}...\Gamma^{\bar{\alpha}_{p_1}})\Gamma_{11}\Big] \quad . \tag{69}
$$

With a simple algebra we see that ζ is symmetric and ζ' is antisymmetric under the exchange of the indices 1 and 2. To see this we use the $(\Gamma^\mu)^T = -C\Gamma^\mu C^{-1}$, $C^T = -C$. Also we have $(\zeta'_{12})^* = \zeta'_{21}$ as needs for the symmetry of amplitude. For $\mathcal{F}_1 = \mathcal{F}_2 = 0$, we have $\zeta' = 0$, therefore, with this special regularization scheme ζ' is purely the effect of the gauge fields. Adding all these together we obtain the contribution of zero modes,

$$
\mathcal{A}^0_{R-R}(\eta_1, \eta_2) = \frac{1}{2}\Big((1 - \eta_1\eta_2)\zeta + (1 + \eta_1\eta_2)\zeta'\Big) \quad . \tag{70}
$$

Note that we can write

$$
C^{-1}G_2^T C = e^{-\frac{1}{2}\mathcal{F}_{(2)\alpha\beta}\Gamma^\alpha\Gamma^\beta} \quad , \tag{71}
$$

with the previous convention for the right hand side. It is worth emphasizing that the right hand side is not G_2^{-1}. Therefore the R-R sector amplitude becomes

$$\begin{aligned}
\mathcal{A}_{R-R} &= \frac{T_{p_1}T_{p_2}}{8(2\pi)^{d_i}}\alpha' \int_0^\infty dt\Big\{\Big(\sqrt{\frac{\pi}{\alpha' t}}\Big)^{d_{in}} e^{-\frac{1}{4\alpha' t}\sum_{i_n}(y_1^{i_n}-y_2^{i_n})^2} \\
&\times \prod_{i_c}\Theta_3\Big(\frac{y_1^{i_c}-y_2^{i_c}}{2\pi R_{i_c}} \mid \frac{i\alpha' t}{\pi(R_{i_c})^2}\Big)\Big[\Big[\zeta\prod_{n=1}^{\infty}\Big[\Big(\frac{1-q^{2n}}{1+q^{2n}}\Big)^2\frac{\det(1+S_1S_2^Tq^{2n})}{\det(1-S_1S_2^Tq^{2n})}\Big]+\zeta'\Big]\Big] \\
&\times \sum_{\{N^{u_c}\}}\Big[(2\pi)^{d_u}\prod_u[\delta(p_1^u-p_2^u)]\ \exp[\frac{i}{\alpha'}\ell^{u_c}(\mathcal{F}_{(1)\ u_c}^{\alpha'_1}y_2^{\alpha'_1}-\mathcal{F}_{(2)\ u_c}^{\alpha'_2}y_1^{\alpha'_2})] \\
&\times \exp[-\frac{t}{\alpha'}\ell^{u_c}\ell^{v_c}(\eta_{u_cv_c}+\mathcal{F}_{(1)\ u_c}^{u}\mathcal{F}_{(2)\ uv_c}+\mathcal{F}_{(1)\ u_c}^{\alpha'_1}\mathcal{F}_{(1)\ v_c}^{\alpha'_1}+\mathcal{F}_{(2)\ u_c}^{\alpha'_2}\mathcal{F}_{(2)\ v_c}^{\alpha'_2})]\Big]\Big\}\ . \quad (72)
\end{aligned}$$

We see that the signs of ζ and ζ' depend on the arrangements of $\bar{\alpha}_i$'s (and $\bar{\beta}_i$'s) in their arguments, therefore the R-R forces may be repulsive or attractive due to the brane-brane or brane-antibrane interaction. Because of our special regularization, the quantity ζ' is usually zero. Due to their procedure, authors of Ref.[7], have non-zero ζ' for $D_0 - D_8$ system. Some special configurations have non zero ζ', for example consider m_2 and m_8-branes along (X^1, X^9) and $(X^1, ..., X^8)$ directions respectively, then

$$\zeta' = -i\mathrm{Tr}\Big(G_1C^{-1}G_2^TC(\Gamma^9...\Gamma^2)\Gamma_{11}\Big)\ , \quad (73)$$

$$G_1 = 1+\mathcal{F}_{(1)01}\Gamma^0\Gamma^1+\mathcal{F}_{(1)09}\Gamma^0\Gamma^9+\mathcal{F}_{(1)19}\Gamma^1\Gamma^9\ , \quad (74)$$

$$C^{-1}G_2^TC = 1-\mathcal{F}_{(2)01}\Gamma^0\Gamma^1+...\ , \quad (75)$$

therefore

$$\zeta' = 32i(\mathcal{F}_{(2)01}-\mathcal{F}_{(1)01})\ . \quad (76)$$

This result also hold for m_3 and m_7-branes along (X^1, X^8, X^9) and $(X^1, ..., X^7)$ directions.

Comparison of (55) and (72) says that the effects of compactification in $\mathcal{A}_{NS-NS}$ and in $\mathcal{A}_{R-R}$ are the same. This is due to the fact that upon compactification only bosonic contribution is modified. When these results are used in case of parallel mixed branes with the same dimension, those terms which contain α'_1 and α'_2 disappear. Again return to the non compact spacetime, therefore

$$\begin{aligned}
\mathcal{A}_{R-R}^{(nc)} &= \frac{T_{p_1}T_{p_2}}{8(2\pi)^{d_i}}\alpha' V_u\int_0^\infty dt\Big\{\Big(\sqrt{\frac{\pi}{\alpha' t}}\Big)^{d_i}e^{-\frac{1}{4\alpha' t}\sum_i(y_1^i-y_2^i)^2} \\
&\times\Big[\zeta\prod_{n=1}^{\infty}\Big[\Big(\frac{1-q^{2n}}{1+q^{2n}}\Big)^2\frac{\det(1+S_1S_2^Tq^{2n})}{\det(1-S_1S_2^Tq^{2n})}\Big]+\zeta'\Big]\ \Big\}\ . \quad (77)
\end{aligned}$$

3.3 A special case

Now we consider the important example of parallel branes with the same $\mathcal{F}$. Consider two parallel m_{p_1} and m_{p_2}-branes which their world-branes are at $(X^0, X^1, ..., X^{p_1})$ and $(X^0, X^1, ..., X^{p_1}, ..., X^{p_2})$ respectively with $\gamma \equiv p_2 - p_1 \geq 0$. Also consider $\mathcal{F}_{(1)\,uv} = \mathcal{F}_{(2)\,uv} \equiv \mathcal{F}_{uv}$ for $u, v \in \{0, 1, ..., p_1\}$ and all other components of $\mathcal{F}_2$ be zero, therefore orthogonality of $Q_{(1)\,uv}(= Q_{(2)\,uv})$ gives

$$\det(1 + S_1 S_2^T q_n) = (1 + q_n)^{10-\gamma}(1 - q_n)^{\gamma} \quad , \tag{78}$$

where $q_n = \pm q^{2n}, \pm q^{2n-1}$. Also equality of the field strengths implies $G_1 = G_2 \equiv G$, therefore

$$\zeta = -\frac{1}{2}\delta_{\gamma,0}\mathrm{Tr}(GC^{-1}G^T C) = -16\delta_{\gamma,0}\det(1 - \mathcal{F}) \quad , \tag{79}$$

the last equality can be investigated for each "p_1" individually. For this special configuration equality of the field strengths implies $\zeta' = 0$. Finally the total amplitude $\mathcal{A} = \mathcal{A}_{NS-NS} + \mathcal{A}_{R-R}$ becomes

$$\begin{aligned}
\mathcal{A} \;=\; & \frac{T_{p_1}T_{p_2}\alpha' V_{p_1+1}}{8(2\pi)^{9-p_2}}\det(1-\mathcal{F})\int_0^\infty dt\Big\{\Big(\sqrt{\frac{\pi}{\alpha' t}}\Big)^{d_{i_n}} e^{-\frac{1}{4\alpha' t}\sum_{i_n}(y_1^{i_n}-y_2^{i_n})^2} \\
& \times \prod_{i_c}\Theta_3\Big(\frac{y_1^{i_c}-y_2^{i_c}}{2\pi R_{i_c}} \mid \frac{i\alpha' t}{\pi (R_{i_c})^2}\Big)\Big(\Big(\frac{1}{q}\Big[\Big[\prod_{n=1}^{\infty}\Big[\Big(\frac{1+q^{2n-1}}{1-q^{2n}}\Big)^{8-\gamma}\Big(\frac{1-q^{2n-1}}{1+q^{2n}}\Big)^{\gamma}\Big] \\
& -\prod_{n=1}^{\infty}\Big[\Big(\frac{1-q^{2n-1}}{1-q^{2n}}\Big)^{8-\gamma}\Big(\frac{1+q^{2n-1}}{1+q^{2n}}\Big)^{\gamma}\Big]\;\Big]\Big] - 16\delta_{\gamma,0}\prod_{n=1}^{\infty}\Big(\frac{1+q^{2n}}{1-q^{2n}}\Big)^{8-2\gamma}\Big)\Big) \\
& \times \sum_{\{N^{u_c}\}}\exp[-\frac{t}{\alpha'}\ell^{u_c}\ell^{v_c}(\eta_{u_c v_c} + \mathcal{F}^{u}{}_{u_c}\mathcal{F}_{u v_c})]\Big\} \quad .
\end{aligned} \tag{80}$$

Therefore the tensions are modified by the factor $\sqrt{\det(1-\mathcal{F})}$. Apart from the modification of the tensions, field strengths $\mathcal{F}$ appear in this interaction amplitude only through the compactification effects. The first two terms come from the NS-NS sector, for $\gamma = p_2 - p_1 = 4$ the NS-NS sector amplitude vanishes. The third term comes from the R-R sector, and show that the amplitude of the R-R sector for the branes of different dimensions ($\gamma \neq 0$) vanishes. For $\gamma = 0$, total amplitude $\mathcal{A}$ vanishes (due to the "abstruse identity") so the BPS no force condition is satisfied.

In non-compact spacetime, making a transformation $t \rightarrow \pi/2t$ and for $T_p = \sqrt{\pi}(4\pi^2\alpha')^{(3-p)/2}$ the last zero amplitude transforms to the known parallel D_p-branes amplitude [20] with the expected extra factor,

$$\begin{aligned}
\mathcal{A} \;=\; & V_{p+1}\det(1-\mathcal{F})\int_0^\infty \frac{dt}{t}\Big\{(8\pi^2\alpha' t)^{-(p+1)/2}e^{-tY^2/2\pi\alpha'}\prod_{n=1}^{\infty}(1-q^{2n})^{-8} \\
& \times\frac{1}{2}\Big[\frac{1}{q}\Big(\prod_{n=1}^{\infty}(1+q^{2n-1})^8 - \prod_{n=1}^{\infty}(1-q^{2n-1})^8\Big) - 16\prod_{n=1}^{\infty}(1+q^{2n})^8\Big]\Big\} \quad ,
\end{aligned} \tag{81}$$

where $q = e^{-\pi t}$ and $Y^i = y_1^i - y_2^i$ is the separation of the branes.

3.4 Other examples

In this part we give the interaction amplitude of the following special systems. In these systems back-ground fields and effects of the compactification appear more explicitly. These systems are : parallel $m_1 - m_{1'}$-branes along X^1, m_1-brane along X^1 perpendicular to $m_{1'}$ along X^2, m_0-brane in front of m_2-brane along X^1X^2, parallel $m_2 - m_{2'}$-branes along X^1X^2, m_2-brane along X^1X^2 perpendicular to $m_{2'}$-brane along X^2X^3, m_1-brane along X^1 parallel to m_5-brane along $X^1...X^5$ directions. For all these we give the following amplitude

$$\begin{aligned}\mathcal{A} \;=\; & \frac{T_p T_{p'} \alpha' V_u}{8(2\pi)^{d_i}} \int_0^\infty dt \Big\{ \Big(\sqrt{\frac{\pi}{\alpha' t}} \Big)^{d_{in}} e^{-\frac{1}{4\alpha' t}\sum_{in}(y_1^{in}-y_2^{in})^2} \prod_{i_c} \Theta_3 \Big(\frac{y_1^{i_c}-y_2^{i_c}}{2\pi R_{i_c}} \mid \frac{i\alpha' t}{\pi (R_{i_c})^2} \Big) \\ & \times \Big(\Big(\sqrt{f f'}\, \frac{1}{q} \Big[\Big[\prod_{n=1}^{\infty} \Big[\Big(\frac{1+q^{2n-1}}{1-q^{2n}} \Big)^N \frac{w(\mathcal{F},\mathcal{F}',q^{2n-1})}{w(\mathcal{F},\mathcal{F}',-q^{2n})} \Big] - \prod_{n=1}^{\infty} \Big[\Big(\frac{1-q^{2n-1}}{1-q^{2n}} \Big)^N \frac{w(\mathcal{F},\mathcal{F}',-q^{2n-1})}{w(\mathcal{F},\mathcal{F}',-q^{2n})} \Big] \Big] \Big] \\ & -16z \prod_{n=1}^{\infty} \Big[\Big(\frac{1+q^{2n}}{1-q^{2n}} \Big)^N \frac{w(\mathcal{F},\mathcal{F}',q^{2n})}{w(\mathcal{F},\mathcal{F}',-q^{2n})} \Big] \Big) \Big) \theta(\mathcal{F},\mathcal{F}',t,R,y) \Big\} \;, \end{aligned} \tag{82}$$

the parameters V_u, d_i, f, f', N and z for the above systems are collected in the following table.

p	p'		V_u	d_i	$\mathcal{F}$	$\mathcal{F}'$	f	f'	N	z
1	1	$\parallel$	$(2\pi R_1)L$	8	$\mathcal{F}_{01} = E$	$\mathcal{F}'_{01} = E'$	$1 - E^2$	$1 - E'^2$	6	$1 - EE'$
1	1	$\perp$	L	7	$\mathcal{F}_{01} = E$	$\mathcal{F}'_{02} = E'$	$1 - E^2$	$1 - E'^2$	5	$-EE'$
2	0	$-$	L	7	$\mathcal{F}_{01} = E_1$ $\mathcal{F}_{02} = E_2$ $\mathcal{F}_{12} = B$	0	$1 - E_1^2$ $-E_2^2 + B^2$	1	5	$-B$
2	2	$\parallel$	$(2\pi R_1)\times$ $(2\pi R_2)L$	7	$\mathcal{F}_{01} = E_1$ $\mathcal{F}_{02} = E_2$ $\mathcal{F}_{12} = B$	$\mathcal{F}'_{01} = E'_1$ $\mathcal{F}'_{02} = E'_2$ $\mathcal{F}'_{12} = B'$	$1 - E_1^2$ $-E_2^2 + B^2$	$1 - E_1'^2$ $-E_2'^2 + B'^2$	5	$1 - E_1E'_1$ $-E_2E'_2 + BB'$
2	2	$\perp$	$(2\pi R_2)L$	6	$\mathcal{F}_{01} = E_1$ $\mathcal{F}_{02} = E_2$ $\mathcal{F}_{12} = B$	$\mathcal{F}'_{02} = E'_2$ $\mathcal{F}'_{03} = E'_3$ $\mathcal{F}'_{23} = B'$	$1 - E_1^2$ $-E_2^2 + B^2$	$1 - E_2'^2$ $-E_3'^2 + B'^2$	4	$E_1E'_3 + BB'$
5	1	$\parallel$	$(2\pi R_1)L$	4	$\mathcal{F}_{01} = E_1$ $\mathcal{F}_{02} = E_2$ $\mathcal{F}_{12} = B$	$\mathcal{F}'_{01} = E'_1$	$1 - E_1^2$ $-E_2^2 + B^2$	$1 - E_1'^2$	2	0

Note that "$\|$" and "$\perp$" stand for the "parallel" and "perpendicular" respectively, and L is infinite time length.

Now we give the functions $\theta(\mathcal{F},\mathcal{F}',t,R,y)$ and $w(\mathcal{F},\mathcal{F}',q_n)$ for these systems, therefore more properties of the interaction of these systems will become clear.

Parallel $\mathbf{m_1}$-branes

For this system we have

$$\theta(E,E',t,R_1)=\frac{2\pi}{L}\sum_{m=-\infty}^{\infty}\delta[(E-E')mR_1/\alpha']e^{-t(1-EE')m^2R_1^2/\alpha'}\quad , \tag{83}$$

where R_1 is the radius of compactification of X^1, therefore $V_2=(2\pi R_1)L$. Also Q is given by the matrix

$$Q=\begin{pmatrix}\frac{1+E^2}{1-E^2} & -\frac{2E}{1-E^2}\\ -\frac{2E}{1-E^2} & \frac{1+E^2}{1-E^2}\end{pmatrix}\quad , \tag{84}$$

and Q' has the same form as Q in which E is replaced by E'. We also have,

$$w(E,E',q_n)=\det(1+q_nQQ'^T)\quad . \tag{85}$$

Note that to get Q'^T from Q', one must use of

$$(Q'^T)^{\alpha}{}_{\beta}=(Q')_{\beta}{}^{\alpha}=\eta^{\alpha\alpha}\eta_{\beta\beta}(Q')^{\beta}{}_{\alpha}\quad , \tag{86}$$

therefore

$$w(E,E',q_n)=\Big(1+\frac{(1-E)(1+E')}{(1+E)(1-E')}q_n\Big)\Big(1+\frac{(1+E)(1-E')}{(1-E)(1+E')}q_n\Big)\quad . \tag{87}$$

For $E=E'$ we have

$$\theta(E,E,t,R_1)=\Theta_3\Big(0\mid\frac{it(1-E^2)R_1^2}{\pi\alpha'}\Big)\quad , \tag{88}$$

therefore through the compactification, fields $E=E'$ appear in the amplitude as in (88) (except for the factors $\sqrt{1-E^2}$ in the modification of the tensions). For $E=E'$ the amplitude vanishes (due to the abstruse identity).

Perpendicular $\mathbf{m_1}$-branes

In this case

$$\theta(E,E',t,R_\alpha)=1\quad , \tag{89}$$

also $w(E,E',q_n)=\det(1+q_n\Omega\Omega'^T)$ where Ω and Ω' are

$$\Omega=\begin{pmatrix}Q & 0\\ 0 & -1\end{pmatrix}\quad , \tag{90}$$

$$\Omega' = \begin{pmatrix} \frac{1+E'^2}{1-E'^2} & 0 & -\frac{2E'}{1-E'^2} \\ 0 & -1 & 0 \\ -\frac{2E'}{1-E'^2} & 0 & \frac{1+E'^2}{1-E'^2} \end{pmatrix} \quad , \tag{91}$$

where Q is the same as in (84). After the expansion of the determinant we see that the function $w(E, E', q_n)$ is symmetric under the exchange of E and E', as expected, For this system $z = EE'$, therefore R-R interaction may be attractive, repulsive or zero, according to the signs and values of E and E'. We remind that the function w simplifies to,

$$w(E, 0, q_n) = (1 - q_n)^2(1 + q_n) \quad . \tag{92}$$

$\mathbf{m_2 - m_0}$ Branes system

For this system the functions θ and w are

$$\theta(\mathcal{F}, \mathcal{F}', t, R_\alpha) = 1 \quad , \tag{93}$$

$$w(E_1, E_2, B, q_n) = \det(1 + q_n Q \Omega'^T) \quad , \tag{94}$$

where Q and Ω' are 3×3 matrices

$$\Omega' = \mathrm{diag}(1,\ -1,\ -1) \quad , \tag{95}$$

$$Q = \frac{1}{f} \begin{pmatrix} (1 + E_1^2 + E_2^2 + B^2) & 2(-E_1 + E_2 B) & -2(E_2 + E_1 B) \\ -2(E_1 + E_2 B) & (1 + E_1^2 - E_2^2 - B^2) & 2(B + E_1 E_2) \\ 2(-E_2 + E_1 B) & 2(-B + E_1 E_2) & (1 - E_1^2 + E_2^2 - B^2) \end{pmatrix} \quad , \tag{96}$$

and $f = 1 - E_1^2 - E_2^2 + B^2$. After the expansion of the determinant we see that $w(E_1, E_2, B, q_n)$ is symmetric under the exchange of the E_1 and E_2 as expected. Also $z = -B$ says that the R-R interaction is attractive for positive $\mathcal{F}_{12} = B$ and is repulsive for negative B, in other word R-R force depends on the fact that m_0-brane is in what sides of m_2-brane.

Parallel $\mathbf{m_2}$-branes

In this case again we can write the function w as $w(\mathcal{F}, \mathcal{F}', q_n) = \det(1 + q_n Q Q'^T)$, where the matrix Q is given in (96). The matrix Q' has exactly the same form of the matrix Q with E_1, E_2 and B changed to E'_1, E'_2 and B' respectively. (We remind that $(Q'^T)^\alpha{}_\beta = \eta^{\alpha\alpha}\eta_{\beta\beta}(Q')^\beta{}_\alpha$).

Now consider the case where X^1 and X^2-directions are both compact. Therefore

$$\theta(\mathcal{F},\mathcal{F}',t,R_1,R_2) = \frac{(2\pi)^3}{V_3}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\Big\{\delta\Big((E_1-E'_1)mR_1/\alpha'+(E_2-E'_2)nR_2/\alpha'\Big)$$
$$\times\delta[(B-B')mR_1/\alpha']\delta[(B-B')nR_2/\alpha']\exp\Big[-\frac{t}{\alpha'}\Big((1-E_1E'_1+BB')m^2R_1^2$$
$$+(1-E_2E'_2+BB')n^2R_2^2-(E_1E'_2+E'_1E_2)mnR_1R_2\Big)\Big]\Big\}\ , \tag{97}$$

where $V_3=(2\pi R_1)(2\pi R_2)L$ is the world volume of the m_2-branes. We see that the functions w and θ are symmetric under the exchange of the fields $\mathcal{F}$ and $\mathcal{F}'$, as expected. Specially consider $E'_1=E_1,\ E'_2=E_2$ and $B'=B$, which give $QQ'^T=1$. Therefore except for the factors $\sqrt{1-E_1^2-E_2^2+B^2}$ in the modification of the tensions, compactification causes that these fields to appear in the amplitude by the equation (97). More specially let $E_2=E'_2=0$ then

$$\theta(E_1,B,t,R_1,R_2)=\Theta_3\Big(0\mid\frac{it(1-E_1^2+B^2)R_1^2}{\pi\alpha'}\Big)\Theta_3\Big(0\mid\frac{it(1+B^2)R_2^2}{\pi\alpha'}\Big)\ . \tag{98}$$

Perpendicular $\mathbf{m_2}$-branes

Consider m_2-brane along the (X^1,X^2) and $m_{2'}$-brane along (X^2,X^3) directions, then $w(\mathcal{F},\mathcal{F}',q_n)=\det(1+q_n\Omega\Omega'^T)$, where Ω and Ω' are

$$\Omega=\begin{pmatrix} Q & 0\\ 0 & -1\end{pmatrix}\ , \tag{99}$$

$$\Omega'=\begin{pmatrix}
\frac{1}{f'}(1+E'^2_2+E'^2_3+B'^2) & 0 & \frac{2}{f'}(-E'_2+E'_3B') & -\frac{2}{f'}(E'_3+E'_2B')\\
0 & -1 & 0 & 0\\
-\frac{2}{f'}(E'_2+E'_3B') & 0 & \frac{1}{f'}(1+E'^2_2-E'^2_3-B'^2) & \frac{2}{f'}(B'+E'_2E'_3)\\
\frac{2}{f'}(-E'_3+E'_2B') & 0 & \frac{2}{f'}(-B'+E'_2E'_3) & \frac{1}{f'}(1-E'^2_2+E'^2_3-B'^2)
\end{pmatrix} \tag{100}$$

where Q is given in (96) and $f'=1-E'^2_1-E'^2_2+B'^2$. The function θ is

$$\theta(E_2,B,E'_2,B',y_1^3,y_2^1,t,R_2)=\frac{2\pi}{L}\sum_{m=-\infty}^{\infty}\Big\{\delta[(E_2-E'_2)mR_2/\alpha']$$
$$\times\exp\Big(\frac{i}{\alpha'}mR_2(By_2^1+B'y_1^3)-\frac{t}{\alpha'}(1-E_2E'_2+B^2+B'^2)m^2R_2^2\Big)\Big\}\ , \tag{101}$$

for $E_2\neq E'_2$ we have $\theta=1$, but for $E_2=E'_2$ it is

$$\theta(E_2,B,E_2,B',y_1^3,y_2^1,t,R_2)=\Theta_3\Big(\frac{(By_2^1+B'y_1^3)R_2}{2\pi\alpha'}\mid\frac{it(1-E_2^2+B^2+B'^2)R_2^2}{\pi\alpha'}\Big)\ . \tag{102}$$

$\mathbf{m_5}$-brane parallel to $\mathbf{m_1}$-brane

For simplicity consider $\mathcal{F}_{01} = E_1,\ \mathcal{F}_{02} = E_2,\ \mathcal{F}_{12} = B$ and all other components of $\mathcal{F}_{\alpha\beta}$ be zero, these with $\mathcal{F}'_{01} = E'_1$ give

$$w(\mathcal{F}, \mathcal{F}', q_n) = (1 - q_n)^3 \det(1 + q_n \Omega \Omega'^T) \ , \tag{103}$$

where Ω is the same as Q in (96) and Ω' is given by (90) in which E must change to E'_1. The function θ is

$$\theta(E_1, B, E'_1, t, R_1) = \frac{2\pi}{L} \sum_{m=-\infty}^{\infty} \delta[(E_1 - E'_1) m R_1/\alpha'] \times \exp\Big(-\frac{i}{\alpha'} m R_1 B y_2^2 - \frac{t}{\alpha'} m^2 R_1^2 (1 - E_1 E'_1 + B^2)\Big) \ . \tag{104}$$

Note that for $E_1 \neq E'_1$ it is equal to 1, and for $E_1 = E'_1$ is given by Jacobi function,

$$\theta(E_1, B, E_1, t, R_1) = \Theta_3\Big(-\frac{R_1 B y_2^2}{2\pi\alpha'} \mid \frac{it(1 - E_1^2 + B^2) R_1^2}{\pi\alpha'}\Big) \ . \tag{105}$$

For $E'_1 = E_1$ and $E_2 = B = 0$, NS-NS interaction vanishes. R-R interaction of this system for any $\mathcal{F}$ and $\mathcal{F}'$ is always zero.

3.5 Massless states contribution to the amplitude

For distant branes only massless states have a considerable contribution on the interaction amplitude . As NS-NS sector and R-R sector massless states have zero momentum numbers and winding numbers, in equations (55) and (72), only the term with $N^{u_c} = 0$ (for all u_c) contributes to these states. In addition we must calculate the following limit

$$\Omega_{NS} \equiv \lim_{q \to 0} \frac{1}{q} \Big\{ \prod_{n=1}^{\infty} \Big[\Big(\frac{1 - q^{2n}}{1 + q^{2n-1}} \Big)^2 \frac{\det(1 + S q^{2n-1})}{\det(1 - S q^{2n})} \Big] - \prod_{n=1}^{\infty} \Big[\Big(\frac{1 - q^{2n}}{1 - q^{2n-1}} \Big)^2 \frac{\det(1 - S q^{2n-1})}{\det(1 - S q^{2n})} \Big] \Big\} \ , \tag{106}$$

for the NS-NS sector and

$$\Omega_R \equiv \lim_{q \to 0} \prod_{n=1}^{\infty} \Big[\Big(\frac{1 - q^{2n}}{1 + q^{2n}} \Big)^2 \frac{\det(1 + S q^{2n})}{\det(1 - S q^{2n})} \Big] \ , \tag{107}$$

for the R-R sector, where $q = e^{-2t}$ and $S = S_1 S_2^T$. For a matrix A we have $detA = e^{Tr[lnA]}$ therefore

$$\prod_{n=1}^{\infty} \Big(\det(1 + q_n S') \Big) = \exp\Big\{ \sum_{k=0}^{\infty} \Big[\frac{(-1)^k Tr(S'^{k+1})}{k+1} \sum_{n=1}^{\infty} q_n^{k+1} \Big] \Big\} \ , \tag{108}$$

where $q_n = q^{2n}$, q^{2n-1} and $S' = \pm S, \pm 1$, thus,

$$\mathcal{A}_0^{(NS-NS)} = \frac{T_{p_1}T_{p_2}}{4(2\pi)^{d_i}}\alpha' V_u \sqrt{\det(1-\mathcal{F}_1)\det(1-\mathcal{F}_2)}\ [Tr(S_1 S_2^T) - 2]$$
$$\times \int_0^\infty dt \Big\{ \Big(\sqrt{\frac{\pi}{\alpha' t}}\Big)^{d_{in}} e^{-\frac{1}{4\alpha' t}\sum_{in}(y_1^{in}-y_2^{in})^2} \prod_{i_c} \Theta_3\Big(\frac{y_1^{i_c}-y_2^{i_c}}{2\pi R_{i_c}} \mid \frac{i\alpha' t}{\pi (R_{i_c})^2}\Big)\Big\} \quad (109)$$

For the system that was considered in subsection 3.3, and for $\gamma = 4$ this vanishes, meaning that the attractive force of graviton and dilaton cancel the repulsive force of Kalb-Ramond field. For this case

$$\mathcal{A}_0^{(R-R)} = \frac{T_{p_1}T_{p_2}}{8(2\pi)^{d_i}}(\zeta+\zeta')\alpha' V_u \int_0^\infty dt \Big\{ \Big(\sqrt{\frac{\pi}{\alpha' t}}\Big)^{d_{in}}$$
$$\times e^{-\frac{1}{4\alpha' t}\sum_{in}(y_1^{in}-y_2^{in})^2} \prod_{i_c} \Theta_3\Big(\frac{y_1^{i_c}-y_2^{i_c}}{2\pi R_{i_c}} \mid \frac{i\alpha' t}{\pi (R_{i_c})^2}\Big)\Big\} \ . \quad (110)$$

Again for the system of subsection 3.3, for $\gamma \neq 0$ this always is zero.

For parallel m_p-branes (or anti m_p-branes) with $\mathcal{F}_1 = \mathcal{F}_2 \equiv \mathcal{F}$ in non compact space time the total massless states amplitude $\Big(\mathcal{A}_0 = \mathcal{A}_0^{(NS-NS)} + \mathcal{A}_0^{(R-R)}\Big)$ is

$$\mathcal{A}_0 = (1-1)V_{p+1} 2T_p^2 G_{9-p}(Y^2)\det(1-\mathcal{F}) \ , \quad (111)$$

where $Y^i = y_1^i - y_2^i$ and $G_D(Y^2)$ is the Green's function in D dimension. For $T_p = \sqrt{\pi}(4\pi^2\alpha')^{(3-p)/2}$, quantity $\mathcal{A}_0$ agrees with the known cases in the literatures with the expected extra factor $\det(1-\mathcal{F})$.

4 Conclusion

We explicitly showed that how the total field strength $\mathcal{F}$ and compactification effects appear in the boundary states. A novel feature is to cause closed string states to have a momentum along the brane, where the branes are wrapped on the compact directions.

We obtained the general form of the amplitude for branes with arbitrary dimensions p_1, p_2 and internal field strengths $\mathcal{F}_1$ and $\mathcal{F}_2$ for both compact and non-compact spaces. The sign of the zero mode part of R-R sector amplitude ζ and ζ' corresponds to the brane-brane (antibrane-antibrane) or brane-antibrane interactions. For parallel mixed branes with the same total field strength, only in the compactified space the field strength appears in the interaction amplitude (except for the factors $\sqrt{\det(1-\mathcal{F})}$ in the modification of the tensions). For this system when $p_2 - p_1 = 4$, the NS-NS interaction vanishes, for $p_1 = p_2$ total interaction amplitude is zero, so the BPS no force condition is satisfied.

Acknowledgement

The authors would like to thank M.M. Sheikh-Jabbari for useful discussion.

References

[1] M.B.Green and P.Wai, Nucl. Phys. **B431** (1994) 131; C. Schmidhuber, Nucl. Phys. **B467** (1996) 146, hep-th/9601003; M. B. Green and M. Gutperle, Nucl. Phys. **B476** (1996) 484, hep-th/9604091.

[2] M. Billo, P. Di Vecchia and D. Cangemi, Phys. Lett.**400B**(1997) 63, hep-th/9701190 .

[3] M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russo, Phys. Lett. **400B**(1997)52, hep-th/9702037 .

[4] F. Hussain, R. Iengo and C. Nunez, Nucl. Phys.**B497**(1997)205, hep-th/9701143 .

[5] O. Bergman, M. Gaberdiel and G. Liftschytz, Nucl. Phys.**B509** (1998)194, hep-th/9705130 .

[6] P. Di Vecchia, M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russo, Nucl. Phys. **B507**(1997)259, hep-th/9707068 .

[7] M. Billo, P. Di Vecchia, M. Frau, A. Lerda, I. Pesando, R. Russo and S. Sciuto, Nucl. Phys.**B526**(1998)199, hep-th/9802088 .

[8] H. Arfaei and M.M. Sheikh Jabbari, Nucl. Phys.**B526**(1998)278, hep-th/9709054 .

[9] M.M. Sheikh Jabbari, Phys. Lett.**B425**(1998)48, hep-th/9712199 .

[10] S. Gukov, I.R. Klebanov and A.M. Polyakov, Phys. Lett.**B423**(1998) 64, hep-th/9711112 .

[11] M. Li, Nucl.Phys.**B460**(1996)351-361 , hep-th/9510161 .

[12] T. Kitao, N. Ohta and J. Ge Zho, Phys. Lett.**B428**(1998)68, hep-th/9801135 .

[13] C.G. Callan, C. Lovelace, C.R. Nappi and S.A. Yost, Nucl. Phys.**B308** (1988)221-284 .

[14] C.G. Callan and I.R. Klebanov, Nucl. Phys.**B465**(1996)473, hep-th/9511173 .

[15] M. Billo, P. Di Vecchia, M. Frau, A. Lerda, R. Russo and S. Sciuto, hep-th/9805091.

[16] H. Arfaei and D. Kamani, Phys. Lett.**B452**(1999)54.

[17] G. Leigh, Mod. Phys. Lett.**A4**28(1989)2767 .

[18] C.G. Callan, C. Lovelace, C.R. Nappi and S.A. Yost, Nucl. Phys.**B288** (1987)525-550.

[19] S.A. Yost, Nucl. Phys.**B321**(1989)629-652 .

[20] J. Polchinski, S. Chaudhuri, C. V. Johnson, hep-th/9602052; J. Polchinski , Phys. Rev. Lett. **75**(1995)4727 .